

A FAST AND ACCURATE CALCULATION SCHEME FOR IONIZATION DEGREES IN PROTOPLANETARY AND CIRCUMPLANETARY DISKS WITH CHARGED DUST GRAINS

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ABSTRACT

We develop a fast and accurate calculation method for ionization degrees in protoplanetary and circumplanetary disks including dust grains. We apply our method to calculate the ionization degree of circumplanetary disks. It is important to understand the structure and evolution of protoplanetary/circumplanetary disks since they are thought to be the sites of planet/satellite formation. The turbulence that causes gas accretion is supposed to be driven by magnetorotational instability (MRI) that occurs only when the ionization degree is high enough for magnetic field to be coupled to gas. We calculate the ionization degrees in circumplanetary disks to estimate the sizes of MRI-inactive regions. We properly include the effect of dust grains because they efficiently capture charged particles and make ionization degree lower. Inclusion of dust grains complicates the reaction equations and requires expensive computation. In order to accelerate the calculation of ionization reactions, we develop a semianalytic method based on the charge distribution model proposed previously. This method enables us to study the ionization state of disks for a wide range of model parameters. For a previous model of circum-Jovian disk, we find that an MRI-inactive region covers almost all regions even without dust grains. This suggests that the gas accretion rates in circumplanetary disks are much smaller than previously thought.

Subject headings: dust, extinction – planets and satellites: formation – protoplanetary disks

1. INTRODUCTION

Various observations suggest substantial gas accretion disks around young stellar objects, so-called protoplanetary disks. Transport of the angular momentum is needed for gas accretion. At present, the magnetohydrodynamic (MHD) turbulence driven by magnetorotational instability (MRI) is thought to be the most promising mechanism for angular momentum transport. However, protoplanetary disks have very low fraction of charged components. This is due to their low temperature and high density; recombination is efficient in high density gas, and thermal ionization does not work except for very inner regions. In order to understand disk evolution, we must clarify which part of the disk is magnetorotationally unstable. Magnetic Reynolds number $Re_m \equiv UL/\eta$ (U and L are characteristic velocity and length respectively, and η is magnetic diffusion coefficient) is used to be a measure of such instability. If Re_m is large, the region is a magnetically active zone; if Re_m is small, the region is a magnetically inactive, so-called, “dead zone” (Gammie 1996). Since η is inversely proportional to the ionization degree, the investigation of ionization degree is important to estimate the value of Re_m , or the location of the MRI-inactive region.

A number of studies on the ionization degree in protoplanetary disks have appeared in the literature (e.g., Sano et al. 2000; Ilgner & Nelson 2006; Okuzumi 2009). These studies assume steady state for reactions and do not consider the time-dependent ionization events. However, some observation has shown that young stars emit X-ray flares whose timescales are order of a day (Wolk et al. 2005). Some of the dynamical timescales (e.g., reconnection) are expected to be very short. These facts indicate that time-dependent calculation is needed to investigate the ionization degree in realistic dynamical environments.

However, it remains difficult to calculate highly time-dependent ionization degree numerically because the network of chemical reactions in the gas of the disks is highly complicated. Some studies on the ionization degree in protoplanetary disks (e.g. Fromang et al. 2002) did not consider the effect of dust grains since the inclusion of their effect complexifies the reactions further; but the effect of dust grains cannot be ignored, since very efficient capture of charged particles by dust grains makes the ionization degree much lower.

There are many studies on the ionization degree of the protoplanetary disks, but no one has calculated that of circumplanetary disks with dust grains yet. It is important to understand the structure and evolution of circumplanetary disks, since the mass accretion through the disk onto the central planet is important in the early formation phases of the disks, and in addition, they might be the sites of satellites formation.

In this work, we develop a fast and accurate time-dependent calculation method for the ionization degree in protoplanetary and circumplanetary disks. We try to reduce the computation time since we want to plug our method into MHD simulations.

We adopt Gaussian distribution approximation for the charge distribution of dust grains (Okuzumi 2009). This approximation decreases the number of equations and allows us to calculate the ionization degree more efficiently especially when the dust grains have wide range of the charge distribution. We use the piecewise exact solution that is developed by Inoue & Inutsuka (2008). In this method, we analytically solve some part of the reaction equation first, and use the solution as an initial condition of time integration of other terms. This solution enables us to calculate with larger time step. Our method can be applied to both circumplanetary disks and protoplanetary disks, and can be conveniently plugged into multi-dimensional MHD codes.

This paper is organized as follows. In Section 2, we describe the chemical reactions in the planetary disks, and derive equations for our calculation. The methods to speed up the calculation are shown. In Section 3, we test our method by calculating the ionization degree in the protoplanetary disks. In Section 4, we apply our method for circumplanetary disks, and discuss the occurrence of MRI. Summary is in Section 5.

2. REACTIONS AND EQUATIONS

2.1. *Reactions*

Gas in the protoplanetary and circumplanetary disks is mostly neutral hydrogen molecules. However, they are thought to be ionized weakly by ionization source such as cosmic rays, X-rays, and ultraviolet radiation. Resultant ionized particles make secondary ions and molecules, which enable further complex reactions. We describe representative reactions.

When hydrogen molecules are ionized



H_2^+ immediately reacts with H_2 to produce H_3^+ :



Reaction between H_3^+ and molecules (e.g., CO) leads to heavier molecular ions:



Molecular ions are destroyed by charge exchange reactions with atomic heavy metals such as Mg:



As long as atomic heavy metals are abundant, dissociative recombination



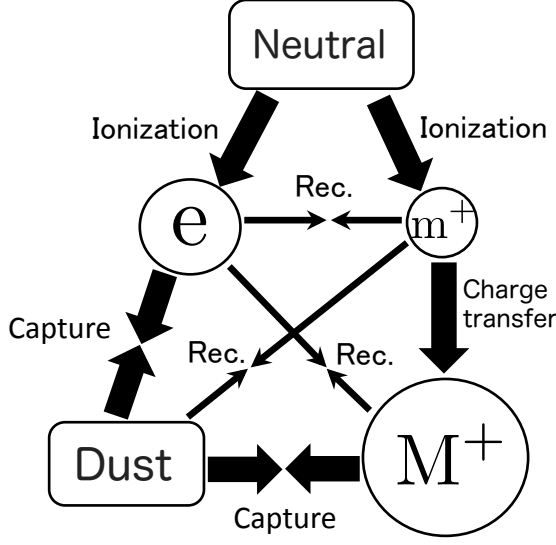


Figure 1. Schematic view of simplified reaction. m^+ , M^+ , and e represent molecular ions, heavy metal ions, and electrons respectively. Neutral gas is ionized by the ionization source, and m^+ transfer their charge to M^+ . Ions and electrons are captured by dust grains, and recombine a little.

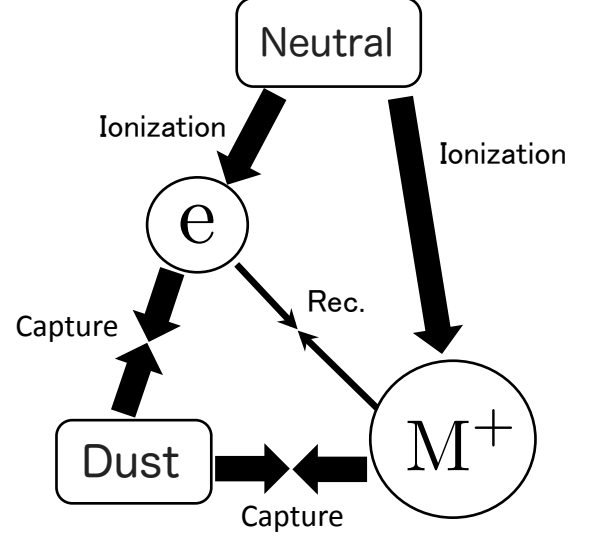


Figure 2. m^+ is removed from Figure 1 because charge transfer is so fast that dissociative recombination and capture by dust grains can be ignored.

is slow, and reaction (Equation (4)) exceeds. A small fraction of metal ions are destroyed by radiative recombination:



We describe major molecular ions (e.g. H_2^+ , H_3^+ , and HCO^+) as m^+ , and major heavy metal ions (e.g. Mg^+ , Fe^+) as M^+ for simplicity (Oppenheimer & Dalgarno 1974; Sano et al. 2000; Ilgner & Nelson 2006).

The effect of dust grains should be also considered. Ions and free electrons are captured by dust grains, and it lowers the ionization degree. With sufficient dust grains, ionization degree is determined mainly by electron capture rate of dust grains. In Figure 1, we show the schematic view of reactions.

2.2. Basic Equations

Most of the gas in a protoplanetary disk is neutral, so we assume that the number density of the neutral gas n_n is large enough and does not depend on ionization degree.

The basic equations are the following:

$$\frac{dn_{m^+}}{dt} = \zeta n_n - \alpha_{m^+} n_{m^+} n_e - \beta n_{m^+} n_{M^+} - \sum_Z k_{m+d}(Z) n_d(Z) n_{m^+}, \quad (7)$$

$$\frac{dn_{M^+}}{dt} = -\alpha_{M^+} n_{M^+} n_e + \beta n_{m^+} n_{M^+} - \sum_Z k_{M+d}(Z) n_d(Z) n_{M^+}, \quad (8)$$

$$\frac{dn_e}{dt} = \zeta n_n - \alpha_{m^+} n_{m^+} n_e - \alpha_{M^+} n_{M^+} n_e - \sum_Z k_{ed}(Z) n_d(Z) n_e, \quad (9)$$

$$\begin{aligned} \frac{dn_d(Z)}{dt} = & -k_{m+d}(Z) n_d(Z) n_{m^+} - k_{M+d}(Z) n_d(Z) n_{M^+} - k_{ed}(Z) n_d(Z) n_e \\ & + k_{m+d}(Z-1) n_d(Z-1) n_{m^+} + k_{M+d}(Z-1) n_d(Z-1) n_{M^+} \\ & + k_{ed}(Z+1) n_d(Z+1) n_e, \end{aligned} \quad (10)$$

where n_j is the number density of each particles ($j = m^+ : \text{molecular ions}, M^+ : \text{heavy metal ions}, M : \text{heavy metal atoms}, e : \text{electrons}, d : \text{dust grains}$), Z is the charge of dust grains, ζ is ionization rate, and α_{m^+} , α_{M^+} , β , and $k_{jd}(Z)$ are the rate coefficients for dissociative recombination, radiative recombination, charge transfer, and capture of gaseous particles by dust grains, respectively. The timescale of charge transfer is so short that we can neglect dissociative recombination and capture of molecular ions by dust grains. This means that almost all the positive charges are transferred to the heavy metal ions. Ignoring the time-derivative term and the second and fourth terms on the right hand side of Equation (7), we obtain the relation $\beta n_{m^+} n_{M^+} \simeq \zeta n_n$. Then, we can write the equations simply as follows:

$$\frac{dn_{M^+}}{dt} = \zeta n_n - \alpha_{M^+} n_{M^+} n_e - \sum_Z k_{M+d}(Z) n_d(Z) n_{M^+}, \quad (11)$$

Reaction	Rate Coefficient [cm ³ /s]
Mg ⁺ + e ⁻ → Mg + hν	α _{M+} = 2.80 × 10 ⁻¹² (T/300) ^{-0.86}
HCO ⁺ + Mg → Mg ⁺ + HCO	β = 2.90 × 10 ⁻⁹

Table 1

Rate coefficients given by UMIST database (RATE'06). T shows the temperature.

$$\frac{dn_e}{dt} = \zeta n_n - \alpha_{M+} n_{M+} n_e - \sum_Z k_{ed}(Z) n_d(Z) n_e, \quad (12)$$

$$\begin{aligned} \frac{dn_d(Z)}{dt} = & -k_{M+d}(Z) n_d(Z) n_{M+} - k_{ed}(Z) n_d(Z) n_e \\ & + k_{M+d}(Z-1) n_d(Z-1) n_{M+} + k_{ed}(Z+1) n_d(Z+1) n_e, \end{aligned} \quad (13)$$

In this way, we can treat the reaction equations as if M⁺ are formed directly by primary ionization as in Figure 2.

Numerical calculation of Equation (11) – (13) is not an easy task. First, as the maximum value of $|Z|$ becomes greater, the number of terms and equations increases, and the system of equations become more complicated. Second, since the timescales in the system are very different, it is difficult to solve the equations explicitly in time. We describe our method to speed up the calculation of these equations in Sections 2.4 and 2.5.

2.3. Rate Coefficients

We use the value of rate coefficients of recombination α and charge transfer β given by UMIST database (RATE'06), and summarize them in Table1. Since we assume that the polarization effect of dust grains is negligible, the rate coefficients of gaseous particle capture by dust grains are written as

$$k_{jd}(Z) \equiv \langle \sigma_{jd}(Z) v_j \rangle_v, \quad (14)$$

where $\langle \rangle_v$ means the value averaged by the Maxwellian distribution, and σ_{jd} is collisional cross section between dust grains and gaseous particles:

$$\sigma_{jd} = \begin{cases} S_j \pi a^2 \left(1 - \frac{2Q_j Q_d}{am_j v^2}\right) & \left(\frac{1}{2} m_j v^2 > \frac{Q_j Q_d}{a}\right) \\ 0 & \left(\frac{1}{2} m_j v^2 < \frac{Q_j Q_d}{a}\right) \end{cases} \quad (15)$$

where Q_j is the charge of ion or electron, Q_d is that of dust grains, and S_j is the sticking probability. We assume $S_j = 1$ in this paper. With Equation (15), Equation (14) can be calculated as follows.

- $j = i$ (positive ion)

$$k_{id}(z) = \begin{cases} \pi a^2 \langle v_i \rangle_v \exp\left(-\frac{q^2 Z}{ak_B T}\right) & (Z > 0) \\ \pi a^2 \langle v_i \rangle_v \left(1 - \frac{q^2 Z}{ak_B T}\right) & (Z < 0) \end{cases} \quad (16)$$

- $j = e$ (electron)

$$k_{ed}(z) = \begin{cases} \pi a^2 \langle v_e \rangle_v \left(1 + \frac{q^2 Z}{ak_B T}\right) & (Z > 0) \\ \pi a^2 \langle v_e \rangle_v \exp\left(\frac{q^2 Z}{ak_B T}\right) & (Z < 0) \end{cases} \quad (17)$$

where q is the charge of an electron, a is the radius of the grains, k_B is the Boltzmann constant, and $\langle v_j \rangle_v$ is the thermal velocity of the particle:

$$\langle v_j \rangle_v = \sqrt{\frac{8k_B T}{\pi m_j}}. \quad (18)$$

As we can see, particle species dependence is only inverse of the square root of mass. Thus, we do not have to pay much attention to specific species as long as we are interested in ionization degree in a dusty environment.

In this study, we consider the compact dust grains with the density $\rho_{\text{grain}} = 3 \text{ g cm}^{-3}$. The mass of a dust grains is $m_{\text{grain}} = (4\pi/3)\rho_{\text{grain}}a^3$, and the number density of dust grains is $n_d = \rho_d/m_{\text{grain}} = f_{\text{dg}}\rho_n/m_{\text{grain}}$, where f_{dg} is the dust-to-gas mass ratio:

$$f_{\text{dg}} = \frac{\rho_d}{\rho_n} \quad (19)$$

We take a and f_{dg} as free parameters.

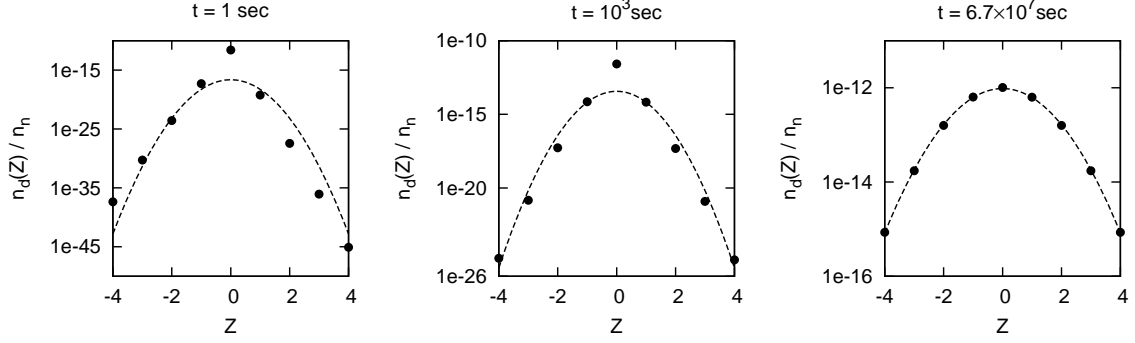


Figure 3. Charge distribution of dust grains. Horizontal axes are charge of dust grains, and vertical axes are the number density of each charge of dust grains normalized by the number density of neutral gas. Dashed lines are fit by the Gaussian distribution. Initial conditions have the dust grains with only $Z = 0$, but the charge distribution already comes close to a Gaussian within only 1 s. At the time ionization degree becomes equilibrium ($t \sim 10^7$ s), charge distribution turns out to be a Gaussian. The middle panel is on the way to be equilibrium.

2.4. Gaussian Approximation for Charge Distribution of Dust Grains

In the basic equations, the third terms of Equation (11) and (12) contain a number of Z -dependent terms. This means that the wider the charge distribution we consider, the more reactions we have to solve. Additionally, Equation (13) is consist of many equations. Though we should consider a wide charge distribution for reality, it takes a long time to deal with such a large amount of calculation.

Okuzumi (2009) has shown that charge distribution of dust grains can be approximated by a Gaussian distribution. We confirmed this by solving the basic equations with $-4 \leq Z \leq 4$. We use $\zeta = 7.6 \times 10^{-19} \text{s}^{-1}$, $T = 280 \text{K}$, $a = 0.1 \mu\text{m}$, and $f_{\text{dg}} = 10^{-2}$. Initial condition is $n_{\text{M}^+} = 0$, $n_{\text{e}} = 0$, $n_{\text{d}}(0) = 1.11 \times 10^3 \text{cm}^{-3}$, and $n_{\text{d}}(Z \neq 0) = 0$. Figure 3 is the plots of the number density of dust grains as a function of charge Z and their fit by the Gaussian distribution. We can see the charge distribution evolves toward a Gaussian distribution. Ionization degree reaches equilibrium by $t \sim 10^7$ s, and at that time the distribution can be well approximated by a Gaussian. On the left panel, dust grains tend to be charged negatively, because electron can be captured by dust grain more quickly than ions; electrons have larger velocity than ions because they are lighter.

We define the total number density of dust grains as

$$N_{\text{d}} \equiv \sum_Z n_{\text{d}}(Z). \quad (20)$$

Since it turns out that charge distribution of dust grains can be approximated by a Gaussian, $n_{\text{d}}(Z)$ can be written as

$$n_{\text{d}}(Z) = \frac{N_{\text{d}}}{\sqrt{2\pi\langle\delta Z^2\rangle}} \exp\left[-\frac{(Z - \langle Z \rangle)^2}{2\langle\delta Z^2\rangle}\right] \quad (21)$$

The number density of charged dust grains should be obtained by calculating the time evolution of the following mean charge of dust grains $\langle Z \rangle$ and dispersion of the distribution $\langle\delta Z^2\rangle$:

$$\langle Z \rangle \equiv \frac{1}{N_{\text{d}}} \sum_Z Z n_{\text{d}}(Z) \simeq \frac{1}{N_{\text{d}}} \int_{-\infty}^{\infty} Z n_{\text{d}}(Z), \quad (22)$$

$$\begin{aligned} \langle\delta Z^2\rangle &\equiv \langle(Z - \langle Z \rangle)^2\rangle \\ &= \langle Z^2 \rangle - \langle Z \rangle^2 \\ &= \frac{1}{N_{\text{d}}} \sum_Z Z^2 n_{\text{d}}(Z) - \langle Z \rangle^2 \\ &\simeq \frac{1}{N_{\text{d}}} \int_{-\infty}^{\infty} Z^2 n_{\text{d}}(Z) - \left[\frac{1}{N_{\text{d}}} \int_{-\infty}^{\infty} Z n_{\text{d}}(Z) \right]^2, \end{aligned} \quad (23)$$

where $\langle \rangle$ means the averaged value weighted by the number density of dust grains $n_{\text{d}}(Z)$.

First, we take the first moment of Equation (13):

$$\begin{aligned} \sum_Z Z \frac{d}{dt} n_{\text{d}}(Z) &= \sum_Z [k_{\text{M}^+\text{d}}(Z) n_{\text{M}^+} n_{\text{d}}(Z) - k_{\text{ed}}(Z) n_{\text{e}} n_{\text{d}}(Z)] \\ &= \left[\sum_Z k_{\text{M}^+\text{d}}(Z) n_{\text{d}}(Z) \right] n_{\text{M}^+} - \left[\sum_Z k_{\text{ed}}(Z) n_{\text{d}}(Z) \right] n_{\text{e}} \end{aligned}$$

$$= \langle k_{M+d} \rangle N_d n_{M+} - \langle k_{ed} \rangle N_d n_e, \quad (24)$$

where $\langle k_{M+d} \rangle$ and $\langle k_{ed} \rangle$ are averaged rate coefficients weighted by $n_d(Z)$, and their derivation is shown in the Appendix. In Okuzumi (2009), the positively charged cases of $k_{jd}(Z)$ are ignored because dust grains tend to be charged negatively as they grow. In this study, positively charged cases are also considered. This is because the inclusion of positively charged cases slightly affects the results. With Equation (24) and

$$\begin{aligned} \frac{d}{dt} \langle Z \rangle &= \frac{d}{dt} \left[\frac{1}{N_d} \sum_Z Z n_d(Z) \right] \\ &= \frac{1}{N_d} \left[\sum_Z Z \frac{d}{dt} n_d(Z) \right], \end{aligned} \quad (25)$$

we can derive the differential equation of $\langle Z \rangle$:

$$\frac{d \langle Z \rangle}{dt} = \langle k_{M+d} \rangle n_{M+} - \langle k_{ed} \rangle n_e. \quad (26)$$

Furthermore, we take the second moment of Equation (13):

$$\begin{aligned} \sum_Z Z^2 \frac{d}{dt} n_d(Z) &= \langle k_{M+d} \rangle N_d n_{M+} + 2 \langle Z k_{M+d} \rangle N_d n_{M+} \\ &\quad + \langle k_{ed} \rangle N_d n_e - 2 \langle Z k_{ed} \rangle N_d n_e, \end{aligned} \quad (27)$$

In a similar way, we obtain the following equation:

$$\begin{aligned} \frac{d}{dt} \langle Z^2 \rangle &= \langle k_{M+d} \rangle n_{M+} + 2 \langle Z k_{M+d} \rangle n_{M+} \\ &\quad + \langle k_{ed} \rangle n_e - 2 \langle Z k_{ed} \rangle n_e. \end{aligned} \quad (28)$$

Finally, we can derive the differential equation of $\langle \delta Z^2 \rangle$:

$$\begin{aligned} \frac{d \langle \delta Z^2 \rangle}{dt} &= \frac{d \langle Z^2 \rangle}{dt} - \frac{d \langle Z \rangle^2}{dt} \\ &= \frac{d \langle Z^2 \rangle}{dt} - 2 \langle Z \rangle \frac{d \langle Z \rangle}{dt} \\ &= (\langle k_{M+d} \rangle + 2 \langle Z k_{M+d} \rangle - 2 \langle Z \rangle \langle k_{M+d} \rangle) n_{M+} \\ &\quad + (\langle k_{ed} \rangle - 2 \langle Z k_{ed} \rangle + 2 \langle Z \rangle \langle k_{ed} \rangle) n_e \\ &= (\langle k_{M+d} \rangle + 2 \langle k_{M+d} \delta Z \rangle) n_{M+} + (\langle k_{ed} \rangle - 2 \langle k_{ed} \delta Z \rangle) n_e. \end{aligned} \quad (29)$$

The formulae of $\langle k_{M+d} \delta Z \rangle$ and $\langle k_{ed} \delta Z \rangle$ are written in Appendix. By this Gaussian approximation, the number of equations can be dramatically reduced.

2.5. Acceleration by Piecewise Exact Solution

The equations we have to solve are as follows:

$$\frac{dn_{M+}}{dt} = \zeta n_n - \alpha_{M+} n_{M+} n_e - \langle k_{M+d} \rangle N_d n_{M+}, \quad (30)$$

$$\frac{dn_e}{dt} = \zeta n_n - \alpha_{M+} n_{M+} n_e - \langle k_{ed} \rangle N_d n_e, \quad (31)$$

$$\frac{d \langle Z \rangle}{dt} = \langle k_{M+d} \rangle n_{M+} - \langle k_{ed} \rangle n_e, \quad (32)$$

$$\begin{aligned} \frac{d \langle \delta Z^2 \rangle}{dt} &= (\langle k_{M+d} \rangle + 2 \langle k_{M+d} \delta Z \rangle) n_{M+} \\ &\quad + (\langle k_{ed} \rangle - 2 \langle k_{ed} \delta Z \rangle) n_e. \end{aligned} \quad (33)$$

We solve our equations partially with piecewise exact solution (Inoue & Inutsuka 2008). This method enables us to solve with large time steps.

First, we split our equations and solve analytically. The solution of

$$\frac{dn_{M+}}{dt} = \zeta n_n - \langle k_{M+d} \rangle N_d n_{M+}, \quad (34)$$

is

$$n_{M^+}(t + \Delta t) = \left(n_{M^+}(t) - \frac{\zeta n_n}{\langle k_{M+d} \rangle N_d} \right) e^{-\langle k_{M+d} \rangle N_d \Delta t} + \frac{\zeta n_n}{\langle k_{M+d} \rangle N_d}, \quad (35)$$

where Δt is the time step of time integration, and the solution of

$$\frac{dn_e}{dt} = \zeta n_n - \langle k_{ed} \rangle N_d n_e \quad (36)$$

is

$$n_e(t + \Delta t) = \left(n_e(t) - \frac{\zeta n_n}{\langle k_{ed} \rangle N_d} \right) e^{-\langle k_{ed} \rangle N_d \Delta t} + \frac{\zeta n_n}{\langle k_{ed} \rangle N_d}. \quad (37)$$

We treat rate coefficients such as $\langle k_{jd} \rangle$ as constants during one step. From Equation (31) – (33),

$$\begin{aligned} \frac{dn_e}{dt} - \frac{dn_{M^+}}{dt} &= \langle k_{M+d} \rangle n_{M^+} N_d - \langle k_{ed} \rangle n_e N_d, \\ &= N_d \frac{d\langle Z \rangle}{dt}, \end{aligned} \quad (38)$$

and with charge conservation,

$$N_d \langle Z \rangle(t) + n_{M^+}(t) - n_e(t) = \text{constant} = 0, \quad (39)$$

$$\langle Z \rangle(t + \Delta t) = \langle Z \rangle(t) + \frac{n_e(t + \Delta t) - n_{M^+}(t + \Delta t)}{N_d} - \frac{n_e(t) - n_{M^+}(t)}{N_d}. \quad (40)$$

The solution of

$$\begin{cases} \frac{dn_{M^+}}{dt} = -\alpha_{M^+} n_{M^+} n_e \\ \frac{dn_e}{dt} = -\alpha_{M^+} n_{M^+} n_e \end{cases} \quad (41)$$

with Equation (39) is

$$\begin{cases} n_{M^+}(t + \Delta t) = \frac{N_d \langle Z \rangle(t)}{(N_d \langle Z \rangle(t) / n_{M^+}(t) + 1) \exp[\alpha_{M^+} N_d \langle Z \rangle(t) \Delta t] - 1} \\ n_e(t + \Delta t) = n_{M^+}(t + \Delta t) + N_d \langle Z \rangle(t). \end{cases} \quad (42)$$

We use these solutions as initial conditions of the time integration of the remaining complex equation:

$$\begin{aligned} \frac{d\langle \delta Z^2 \rangle}{dt} &= (\langle k_{M+d} \rangle + 2\langle k_{M+d} \delta Z \rangle) n_{M^+} \\ &\quad + (\langle k_{ed} \rangle - 2\langle k_{ed} \delta Z \rangle) n_e. \end{aligned} \quad (43)$$

In conclusion, what to be solved numerically are Equations (35), (37), (40), (42), and (43).

3. TEST CALCULATION ON PROTOPLANETARY DISKS

We compare the result of our fast calculation method with the result of the direct calculation of the original equations (11), (12), and (13) in order to find out how accurate our method is.

3.1. Disk Property of Protoplanetary Disks

We adopt the disk property of the minimum mass solar nebula (MMSN) model (Hayashi 1981). The surface density and the temperature of the gas in the disk are

$$\Sigma_n = 1.7 \times 10^3 \left(\frac{r}{1 \text{ AU}} \right)^{-1.5} \text{ g cm}^{-2}, \quad (44)$$

$$T = 280 \left(\frac{r}{1 \text{ AU}} \right)^{-0.5} \text{ K}, \quad (45)$$

where r is the orbital radius. The density of the disk is defined as follows:

$$\rho_n(r, z) \equiv \frac{\Sigma_g}{\sqrt{2\pi}H} \exp\left(-\frac{z^2}{2H^2}\right), \quad (46)$$

where H is the scale height of the disk:

$$H \equiv \frac{c_s}{\Omega_k}, \quad (47)$$

where $c_s = \sqrt{\gamma k_B T / \mu m_H}$ is the sound speed, $\Omega_k = \sqrt{GM_*/r^3}$ is the Keplerian frequency. Here, γ is the specific heat ratio, μ is the mean molecular weight of the neutral gas, m_H is the mass of a hydrogen atom, G is the gravitational constant, and M_* is the mass of the central star. We adopt $f_{dg} = 10^{-2}$, $\mu = 2.34$, and $M_* = M_\odot$.

3.2. Ionization Rate

There are various primary ionization sources, for example, Galactic cosmic rays, UV and X-rays from central stars, decay of radionuclide, and thermal ionization, etc. Here, just simplicity, we neglect the possible contribution of energetic electrons (Inutsuka & Sano 2005), and only consider cosmic rays, X-rays, and radionuclide. With orbital radius r and perpendicular oriented length z , ionization rate is written as follows:

$$\zeta(r, z) \simeq \zeta_C + \zeta_X + \zeta_R, \quad (48)$$

where ζ_C , ζ_X , and ζ_R are the ionization rate by cosmic rays, stellar X-rays, and radionuclide, respectively. ζ_C is given by the following equation:

$$\zeta_C \simeq \frac{\zeta_{CR}}{2} \left\{ \exp \left[-\frac{\chi(r, z)}{\chi_{CR}} \right] + \exp \left[-\frac{\Sigma(r) - \chi(r, z)}{\chi_{CR}} \right] \right\}, \quad (49)$$

where $\zeta_{CR} \sim 1.0 \times 10^{-17} \text{ s}^{-1}$ is the ionization rate by cosmic rays in the interstellar space, and $\chi_{CR} \sim 96 \text{ g cm}^{-2}$ is the attenuation length of the ionization rate by cosmic rays.

$$\chi(r, z) = \int_z^\infty \rho(r, z) dz \quad [\text{g cm}^{-2}], \quad (50)$$

is the vertical column density of the gas measured from the outside of the disk (Umebayashi & Nakano 1981). The ionization rate by X-ray is given as follows:

$$\zeta_X \simeq \zeta_{XR} \left(\frac{r_*}{1 \text{ AU}} \right)^{-2} \left(\frac{L_{XR}}{2 \times 10^{30} \text{ erg s}^{-1}} \right) \left\{ \exp \left[-\frac{\chi(r, z)}{\chi_{XR}} \right] + \exp \left[-\frac{\Sigma(r) - \chi(r, z)}{\chi_{XR}} \right] \right\}, \quad (51)$$

where r_* is the distance from the star, $L_{XR} \sim 2 \times 10^{30} \text{ erg s}^{-1}$ is the X-ray luminosity, and $\zeta_{XR} \sim 2.6 \times 10^{-15} \text{ s}^{-1}$ and $\chi_{XR} \sim 8.0 \text{ g cm}^{-2}$ are the fitting parameters (Igea & Glassgold 1999; Turner & Sano 2008). We adopt ionization rate by radionuclide $\zeta_R = 7.6 \times 10^{-19} \text{ s}^{-1}$ (Umebayashi & Nakano 2009).

3.3. Result of the Test Calculation

We assume cosmic rays, stellar X-rays, and radionuclide as ionization sources, and calculate at the mid-plane of the orbital radius 1 AU from the central star. The number density of neutral gas is $n_n = \rho_n(1 \text{ AU}, 0) / \mu m_H = 4.22 \times 10^{14} \text{ cm}^{-3}$, and the ionization rate is $\zeta = \zeta(1 \text{ AU}, 0) \simeq 7.6 \times 10^{-19} \text{ s}^{-1}$. We assume the disk is isothermal in the vertical direction, and use $\gamma = 1$ in the expression of c_s .

Figure 4 shows the result of the comparison, and we can see that they agree very well. With our method, we can calculate with about 10^5 times larger time steps. This means that calculation becomes dramatically fast.

We start our calculation with the initial condition $x_{M^+} = 0$, $x_e = 0$, and $\langle Z \rangle = 0$. It takes about 10^7 s to come to be equilibrium. This is the timescale for charged particles to meet dust grains. Okuzumi (2009) has shown that this timescale is written as

$$t^{-1} \simeq \frac{\zeta n_n}{n_d} \quad [\text{s}^{-1}]. \quad (52)$$

The existence of such long timescale events indicates that our time-dependent method is useful.

4. IONIZATION DEGREE IN CIRCUMPLANETARY DISKS

Circum-planetary disks are formed after planet formation and thought to be the site of satellite formation (e.g., Canup & Ward 2002; Sasaki et al. 2010). Understanding of the evolutions of circumplanetary disks is important in the context of satellite formation. One of the important factors in disk evolution is the ionization degree which has been calculated by Takata & Stevenson (1996), although, the surface density used in their study seems to be too heavy, and the effect of dust grains are not concerned. In this Section, we calculate the ionization degree in circumplanetary disks including the charged particle capture by dust grains.

4.1. Disk Property of Circumplanetary Disks

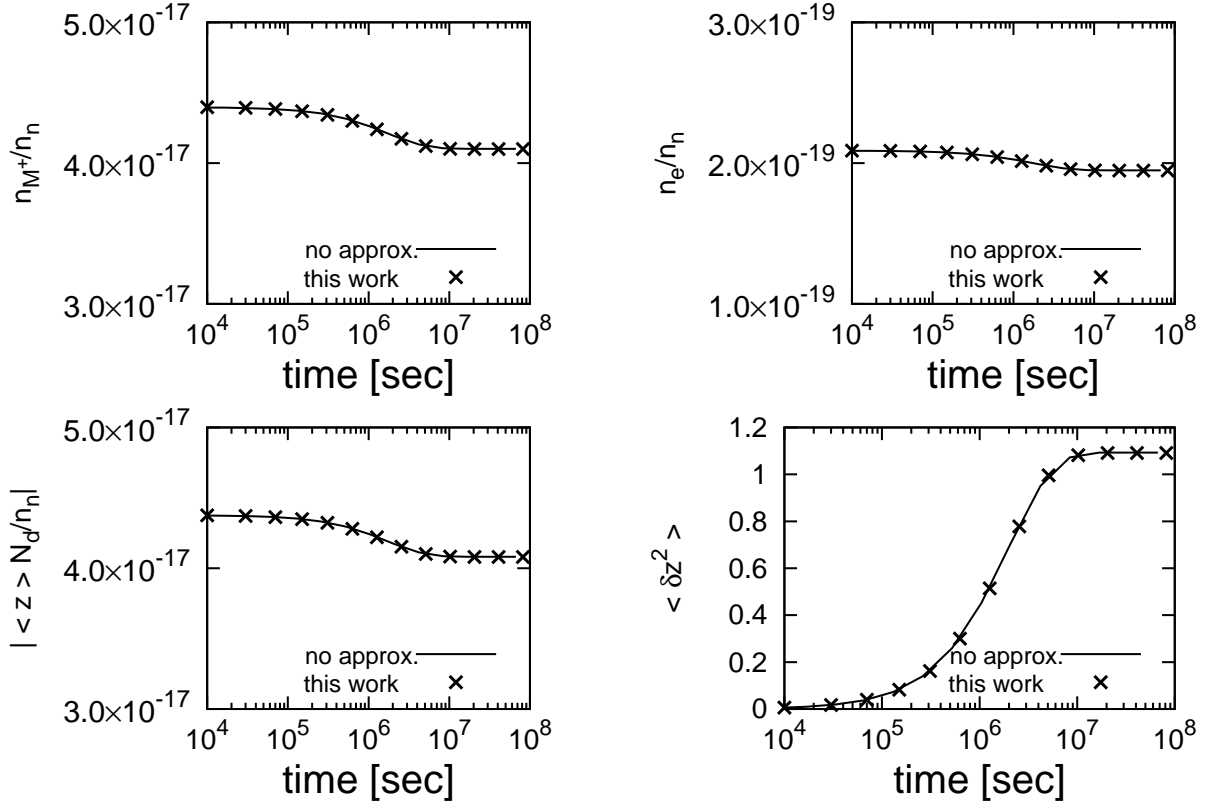


Figure 4. Top-left panel is the number density of heavy metal ion, top-right panel is that of free electron, and both are normalized by the number density of neutral gas. The bottom-left panel is the absolute value of the product of averaged dust charge and the total abundance of dust grains—this means the total charge which dust grains have; and the bottom-right panel is the dispersion of the dust charge distribution. Lines show the results of the basic equations, and crosses show the results of our calculation with two kinds of speed-up device. These panels show that the Gaussian distribution approximation and piecewise exact solution are successful.

We use an actively supplied gaseous accretion disk model (Canup & Ward 2002, 2006; see Appendix of Sasaki et al. 2010). The temperature is

$$T_{\text{cir}} \simeq 160 \left(\frac{M_p}{M_J} \right)^{1/2} \left(\frac{r}{20R_J} \right)^{-3/4} \text{ K}, \quad (53)$$

where M_p is the mass of the central planet, M_J is that of Jupiter, r is the orbital radius from the central planet, and R_J is the radius of Jupiter. In their model, we adopt 5×10^6 yr as the accretion timescale for gaseous disk as heavy as the central planet, and assume the disk is vertically isothermal. The surface density is given as follows with viscosity coefficient $\alpha = 5 \times 10^{-3}$:

$$\Sigma_{\text{cir}} \simeq 100 \left(\frac{M_p}{M_J} \right) \left(\frac{r}{20R_J} \right)^{-3/4} \text{ g cm}^{-2}. \quad (54)$$

4.2. Result

We apply our method for the calculation of the ionization degree in the circumplanetary disks. We assume $M_p = M_J$. The abundance of radionuclide is uncertain, but ionization by cosmic rays is efficient, and radionuclide dose not affect the ionization degree very much. Since circumplanetary disks are located at orbital radius of gas giant planet which is not so close to the star, X-ray ionization is less effective. Furthermore, the scale height of circumplanetary disks is far smaller than that of protoplanetary disks. So, geometrically, it is difficult for X-rays to reach the circumplanetary disks. When we think about circumplanetary disks, we consider only cosmic ray ionization.

In this study, we investigate the extent of the MRI-inactive regions in the circumplanetary disks. We parameterize the vertical component of plasma beta (at mid-plane), the ratio of gas pressure and magnetic pressure, $\beta = P_{\text{gas}}/P_{\text{mag}}$, the radius of dust grains a , and the dust-to-gas mass ratio $f_{\text{dg}} = \rho_d/\rho_g$. Figure 5 shows the size of the MRI-inactive region of the model $\beta = 100$, $a = 10^{-3}$, and $f_{\text{dg}} = 10^{-2}$. As a matter of convenience for the discussion of the MRI, we take z/H as y axes. Figures 6 and 7 show the extent of the MRI-inactive regions. We use Elsasser number as magnetic Reynolds number Re_m :

$$Re_m = \frac{v_{Az}^2}{\eta \Omega_K}, \quad (55)$$

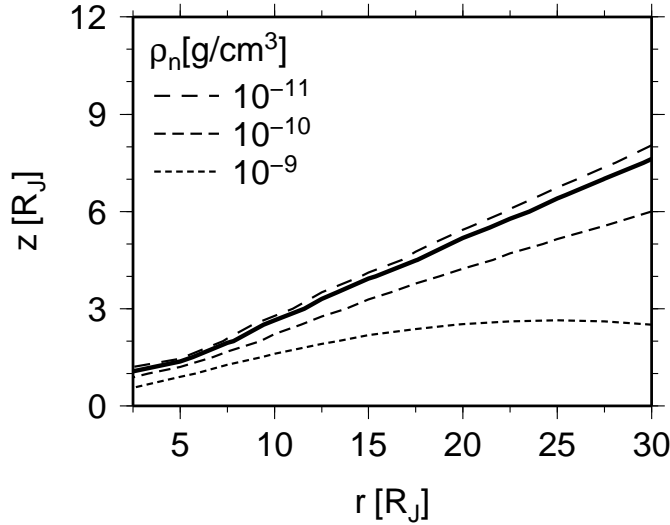


Figure 5. Size of the MRI-inactive region with $\beta = 100$, $a = 10^{-3}$ cm, and $f_{dg} = 10^{-2}$. The horizontal axis is orbital radius from the central planet and vertical axis is vertical extent of the disk. The region under the black line is the MRI-inactive region. The density of the neutral gas is also shown as contour lines.

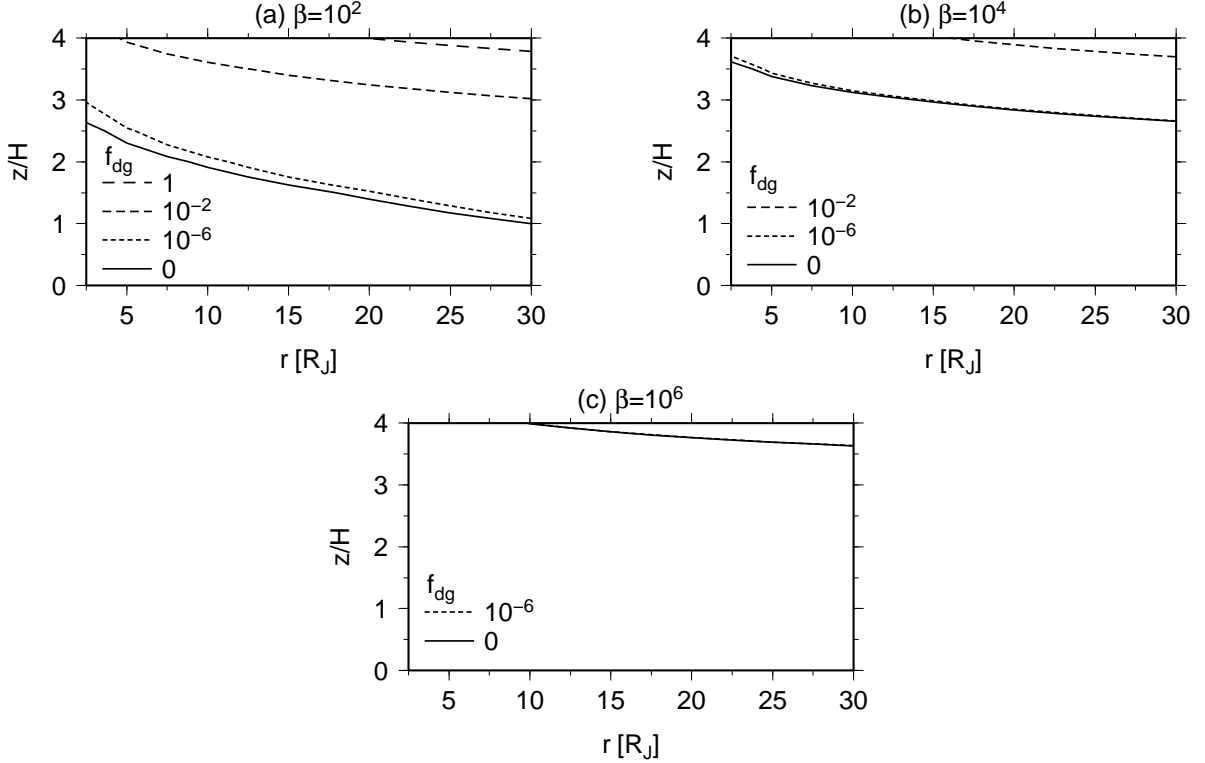


Figure 6. Boundaries of the MRI-inactive region in circumplanetary disks: (a) $\beta = 10^2$, (b) $\beta = 10^4$, and (c) $\beta = 10^6$. The horizontal axis denotes the orbital radius from the central planet, and the vertical axis is vertical extent of the disk that is normalized by the scale height of the corresponding radius. The radius of dust grains is $a = 10 \mu\text{m}$. Contour lines show the boundaries of the MRI-active region and the MRI-inactive region for each model of the dust-to-gas mass ratio f_{dg} . The circumplanetary disks are not expected to be magnetically active.

where v_{Az} is the vertical component of Alfvén velocity. If $Re_m < 1$, MRI does not happen.

In Figure 6, we can see the f_{dg} dependence of the size of the MRI-inactive regions. Figure 7 shows the size of the MRI-inactive regions of various model of a . We also calculate the models of $f_{dg} = 10, 100$. The result of the model $a = 1 \text{ mm}$ for various value of f_{dg} is summarized in Table 2. Our method can calculate even such large mean charge and dispersion of dust grains.

Our result indicates that almost entire parts of circumplanetary disks are MRI-inactive.

4.3. Discussion

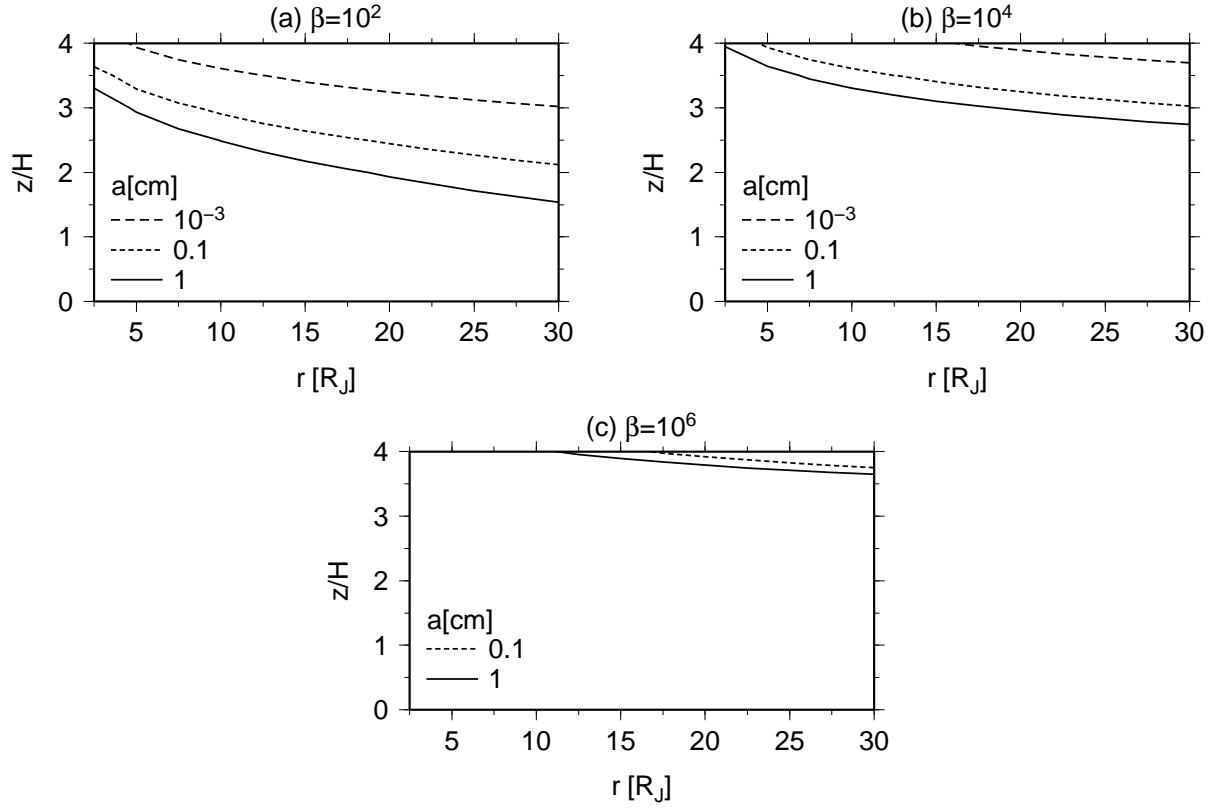


Figure 7. Same as Figure 6, but the dust-to-gas mass ratio is fixed to $f_{dg} = 10^{-2}$, and consider the dust grain radius a cm as a parameter.

f_{dg}	n_e/n_n	$\langle Z \rangle$	$\langle \delta Z^2 \rangle$
100	4.83×10^{-19}	-2.26×10^3	6.46×10^3
10	8.97×10^{-17}	-3.70×10^4	9.56×10^3
1	1.75×10^{-15}	-4.49×10^4	9.83×10^3
10^{-2}	1.76×10^{-13}	-4.50×10^4	9.83×10^3

Table 2

Ionization degree n_e/n_n , mean charge of dust grains (normalized by the charge of an electron) $\langle Z \rangle$, and dispersion of the charge distribution of dust grains $\langle \delta Z^2 \rangle$ at $r = 15R_J$, $z = 0$. The radius of dust grains is fixed at $a = 1$ mm and take dust to gas mass ratio f_{dg} as a parameter. Our method is effective even if the charge distribution of dust grains is very wide.

Previous investigation on circumplanetary disks is mainly focused on active disks for theoretical reasons, but all regions in circumplanetary disks we investigated turn out to be the MRI-inactive region. This suggests that MRI does not occur for a long time, and satellites can be formed slowly, perhaps by gravitational collapse. The temperature of the disk might be lower because viscous heating is not so efficient. However, there are many other accretion mechanisms such as gravitational torques and spiral waves (e.g., Machida et al. 2010). We would like to consider these effects in the future work.

There remains various uncertainty on circumplanetary disks, for example, how many dust grains remain after planet formation, and how large they are. We have to study ionization degree in various cases. We also have to investigate it using other disk models. In this work, we suppose only fixed size of dust grains, so the result may change if we consider the coagulation of dust grains; or if we consider ice as dust grains, result also may change.

5. SUMMARY

We have developed a fast and accurate method to calculate the ionization degree in protoplanetary and circumplanetary disks. This method can calculate it time dependently by using the following two kinds of speed-up device.

1) Gaussian approximation for the charge distribution of dust grains. We have confirmed that the charge distribution of dust grains can be approximated by Gaussian distribution (Okuzumi 2009) by solving basic reaction equations. This approximation can reduce the number of equations.

2) Piecewise exact solution. We have used piecewise exact solution that is developed by Inoue & Inutsuka (2008). We solve only rapid reaction terms analytically in advance, then we solve remaining terms numerically using the analytic solutions as an initial condition of time integration. This method enables us to calculate without limitation of time step by rapid reactions. Since our method can calculate the ionization degree accurately and very rapidly, we would

like to plug our method into MHD simulations.

We have checked our calculation method by comparing the result with the direct calculation of basic reaction equations and shown that they agree very well.

We have applied our calculation method for circumplanetary disks. Our method can calculate ionization degree quickly even when the dispersion of dust grains is about 10^4 . We have investigated with model parameters $a = 10^{-5}, 10^{-3}, 0.1, 1$ cm, $f_{\text{dg}} = 10^{-6}, 10^{-2}, 1, 10, 100$, and $\beta = 10^2, 10^4, 10^6$. The results show that almost all regions of circumplanetary disks are the MRI-inactive regions. This suggests that gas in circumplanetary disks accrete more slowly than previously thought.

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APPENDIX

DERIVATION OF RATE COEFFICIENTS

From Equation (16), the rate coefficient of the metal ion capture by dust grains averaged by charge of dust grains is defined as follows:

$$\begin{aligned}
 \langle k_{\text{M+d}} \rangle &\equiv \frac{1}{N_d} \sum_Z k_{\text{M+d}}(Z) n_d(Z) \\
 &\simeq \frac{1}{N_d} \pi a^2 \langle v_{\text{M+}} \rangle_v \left\{ \int_{-\infty}^0 \left(1 - \frac{q^2 Z}{ak_B T} \right) \frac{N_d}{\sqrt{2\pi \langle \delta Z^2 \rangle}} \exp \left[-\frac{(Z - \langle Z \rangle)^2}{2 \langle \delta Z^2 \rangle} \right] dZ \right. \\
 &\quad \left. + \int_0^{\infty} \exp \left[-\frac{q^2 Z}{ak_B T} \right] \frac{N_d}{\sqrt{2\pi \langle \delta Z^2 \rangle}} \exp \left[-\frac{(Z - \langle Z \rangle)^2}{2 \langle \delta Z^2 \rangle} \right] dZ \right\} \\
 &= \pi a^2 \langle v_{\text{M+}} \rangle_v \left\{ \frac{1}{2} \left(1 - \frac{q^2}{ak_B T} \langle Z \rangle \right) \text{erfc} \left[\frac{\langle Z \rangle}{\sqrt{2 \langle \delta Z^2 \rangle}} \right] \right. \\
 &\quad \left. + \frac{q^2}{ak_B T} \sqrt{\frac{\langle \delta Z^2 \rangle}{2\pi}} \exp \left[-\frac{\langle Z \rangle^2}{2 \langle \delta Z^2 \rangle} \right] \right. \\
 &\quad \left. + \frac{1}{2} \text{erfc} \left[\sqrt{\frac{\langle \delta Z^2 \rangle}{2}} \frac{q^2}{ak_B T} - \frac{\langle Z \rangle}{\sqrt{2 \langle \delta Z^2 \rangle}} \right] \exp \left[\frac{1}{2} \left(\frac{q^2}{ak_B T} \right)^2 \langle \delta Z^2 \rangle - \left(\frac{q^2}{ak_B T} \right) \langle Z \rangle \right] \right\}, \quad (\text{A1})
 \end{aligned}$$

In the same way, we can gain the rate coefficient of electron capture averaged by Z from Equation (17) as the following:

$$\langle k_{\text{ed}} \rangle \simeq \langle k_{\text{M+d}} \rangle |_{\langle Z \rangle \rightarrow -\langle Z \rangle} \quad (\text{A2})$$

We show only the result of derivation of $\langle k_{\text{M+d}} \delta Z \rangle$ and $\langle k_{\text{ed}} \delta Z \rangle$ as follows:

$$\begin{aligned}
 \langle k_{\text{M+d}} \delta Z \rangle &\simeq \pi a^2 \langle v_{\text{M+}} \rangle_v \frac{q^2}{2ak_B T} \langle \delta Z^2 \rangle \left(-\text{erfc} \left[\frac{\langle Z \rangle}{\sqrt{2 \langle \delta Z^2 \rangle}} \right] \right. \\
 &\quad \left. - \exp \left[-\frac{q^2}{ak_B T} \langle Z \rangle + \frac{1}{2} \left(\frac{q^2}{ak_B T} \right)^2 \langle \delta Z^2 \rangle \right] \right. \\
 &\quad \left. \times \text{erfc} \left[\frac{q^2}{ak_B T} \sqrt{\frac{\langle \delta Z^2 \rangle}{2}} - \frac{\langle Z \rangle}{\sqrt{2 \langle \delta Z^2 \rangle}} \right] \right), \quad (\text{A3})
 \end{aligned}$$

$$\begin{aligned}
 \langle k_{\text{ed}} \delta Z \rangle &\simeq \pi a^2 \langle v_e \rangle_v \frac{q^2}{2ak_B T} \langle \delta Z^2 \rangle \left(\text{erfc} \left[-\frac{\langle Z \rangle}{\sqrt{2 \langle \delta Z^2 \rangle}} \right] \right. \\
 &\quad \left. + \exp \left[\frac{q^2}{ak_B T} \langle Z \rangle + \frac{1}{2} \left(\frac{q^2}{ak_B T} \right)^2 \langle \delta Z^2 \rangle \right] \right. \\
 &\quad \left. \times \text{erfc} \left[\frac{q^2}{ak_B T} \sqrt{\frac{\langle \delta Z^2 \rangle}{2}} + \frac{\langle Z \rangle}{\sqrt{2 \langle \delta Z^2 \rangle}} \right] \right). \quad (\text{A4})
 \end{aligned}$$

REFERENCES

- Canup, R. M., & Ward, W. R., 2002, *AJ*, 124, 3404
 Canup, R. M., & Ward, W. R., 2006, *Nature*, 441, 834
 Fromang, S., Terquem, C., & Balbus, S. A., 2002 *MNRAS*, 329, 18
 Gammie, C. F., 1996 *ApJ*, 457, 355
 Hayashi, C., 1981, *Prog. Theor. Phys. Suppl.*, 70, 35
 Igea, J., & Glassgold, A. E., 1999, *ApJ*, 528, 848
 Ilgner, M., & Nelson, R. P., 2006, *A&A*, 445, 205
 Inoue, T., & Inutsuka, S., 2008, *ApJ*, 687, 303
 Inutsuka, S., & Sano, T., 2005, *ApJ*, 628, L155
 Machida, M. N., Kokubo, E., Inutsuka, S., & Matsumoto, T., 2010, *MNRAS*, 405, 1227
 Okuzumi, S. 2009, *ApJ*, 698, 1122
 Oppenheimer, M., & Dalgarno, A., 1974, *ApJ*, 192, 29
 Turner, N. J., & Sano, T., 2008, *ApJ*, 679, L131
 Sano, T., Miyama, S. M., Umebayashi, T., & Nakano, T., 2000, *ApJ*, 543, 486
 Sasaki, T., Stewart, G. R., & Ida, S., 2010 *ApJ*, 714, 1052
 Takata, T., & Stevenson, D. J., 1996, *Icarus*, 123, 404
 Umebayashi, T., & Nakano, T., 1981, *PASJ*, 33, 617
 Umebayashi, T., & Nakano, T., 2009, *ApJ*, 690, 69
 Wolk, S. J., Harnden, F. R., & Flaccomio, E., et al. 2005, *ApJS*, 160, 423